Bonus Chapter 3 — The Math Module

Introduction

Python has many mathematical operations built in, but to perform advanced mathematics we need to import the math module and use its properties and methods. This introduction only covers a few of the features of the math module. See the on-line documentation at https://docs.python.org/3/library/math.html for additional features.

Objectives

Upon completion of this chapter's exercises, you should be able to:

- Blah de blah.
- Baz and Barf.

Prerequisites

This chapter can be included at any point after Chapter 1.

Math

The math module adds many mathematical functions and constants to Python. This introduction will show some of the most widely used methods and constants.

`math`	module
The math module. https://docs.python.org/3/library/math.html	

`import math`	statement
Tell your Python program that it will be using the `math` module. https://docs.python.org/3/library/math.html	

Mathematical Constants

The math module includes several constant values for you to use in your programs. They make programs more consistent using a standard value and by naming these values.

math.pi **math.e**	Literals from math
Returns constant approximate values for the irrational numbers Π (3.1415...) and e (2.71828). The values are approximate to the limits of precision of a floating-point number. These constant values should be sufficient for most purposes. https://docs.python.org/3/library/math.html#constants	

```
1| import math
2|
3| r = 9
4| a = math.pi * r ** 2
5| print("The area of a circle with radius", r, "is", a)
```

```
The area of a circle with radius 9 is 254.46900494077323
```

Floating-Point to Integer

The math module has two methods that will convert a floating-point number to an integer: one is called `floor` and the other `ceil`. The `floor` method will return the integer that is less than or equal to the floating-point number, and `ceil` will return the closest integer greater than or equal to the original number.

```
1| import math
2|
3| a = 10.5
4| print(math.floor(a))
5| print(math.ceil(a))
6|
7| a = -10.5
```

```
8| print(math.floor(a))
9| print(math.ceil(a))
```

```
10
11
-11
-10
```

math.floor(*expression***)** **math.ceil(***expression***)**	Methods of math
Methods commonly used to convert floating-point numbers into integers. The floor method returns the closest whole number that is less than and ceil method returns the closest whole number that is greater than. https://docs.python.org/3/library/math.html#math.ceil https://docs.python.org/3/library/math.html#math.floor	

Logarithms

Logarithms are a way to express a positive real number as the exponent of another real number. You can also think of a logarithm as the inverse operation of exponentiation. They were introduced in the 1600 by John Napier as a way to simplify multiplication and exponentiation.

$x = \log_b(a)$ then $a = b^x$

In simple terms the sum of two logarithms is the same as the logarithm of their products:

$\log(a) + \log(b) = \log(a * b)$ $\log(a) * b = \log(a^b)$

Python can use any real number as a base for logarithms, but natural (base e) and common (base 10) logarithms are used in most calculations.

Base e – Natural Logarithms

Natural logarithms are calculated using a base of e, known as Euler's number, and more formerly known as Napier's constant. It can be found by calculating the infinite sum of $e = 1 + \sum_{n=1}^{\infty} \frac{1}{n!}$ or the

limit at infinity of $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. In reality this number is close to 2.718281828459045. The number e is like pi in that it is an irrational number with an infinite number of digits.

```
1| import math
2|
3| a = math.log(100)
4| print("the natural logarithm of 100 is",a)
5| b = math.exp(a)
6| print("e **", a, "=", b)
```

```
the natural logarithm of 100 is 4.605170185988092
e ** 4.605170185988092 = 100.00000000000004
```

math.log(expression) **math.exp(expression)**	Methods of math
Calculate the natural logarithm of a positive real number or convert a natural logarithm back, by returning e**n. https://docs.python.org/3/library/math.html#power-and-logarithmic-functions	

Base 10 – Common Logarithms

When working with logarithms, especially when making manual calculations or communicating, the base of 10 is often used. It is known as the "Common logarithm".

```
1| import math
2|
3| a = 92193123
4| n = math.log10(a)
5| print("the common logarithm of", a, "is", n)
6| print(a, "is", math.ceil(n), "digits long.")
```

```
the common logarithm of 92193123 is 7.964698526752748
92193123 is 8 digits long.
```

math.log10(expression) **math.pow(mantissa, expression)**	Methods of math
Calculate the common (base 10) logarithm of a positive real number.	

https://docs.python.org/3/library/math.html#power-and-logarithmic-functions

Trigonometry

Trigonometry is the mathematics of triangles. More specifically the relationship between the lengths of sides of a right triangle and the angles.

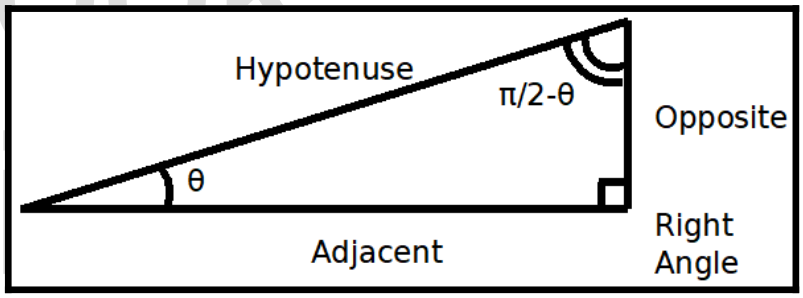

Illustration 38: A Right Triangle

Let us look at a triangle and it's parts. The triangle has three angles: 1) a right angle, 2) the angle we care about (named theta θ), and 3) the the remaining angle (which has the angle of $\frac{\pi}{2}-\theta$ or $90^\circ-\theta$). The three sides are also named: 1) adjacent, that runs from the angle theta to the right angle; 2) opposite, that is directly opposite the angle theta; and 3) the hypotenuse, that runs opposite the right angle.

If you know the length of any two sides of a right triangle, you can find the length of the remaining side by using Pythagoras's Relation. $a^2+b^2=c^2$ where a and b are the lengths of the opposite and adjacent sides, and c is the length of the hypotenuse.

Degrees and Radians to Measure Angles

By default, Python uses radians ($0 \geq 2\pi$) to measure angles. Radians are based on the distance around a unit circle (r=1) and angles with their vertex at the center of that circle. They are very common in mathematics ans science, but most humans use degrees ($0^\circ \geq 360^\circ$) to measure angles.

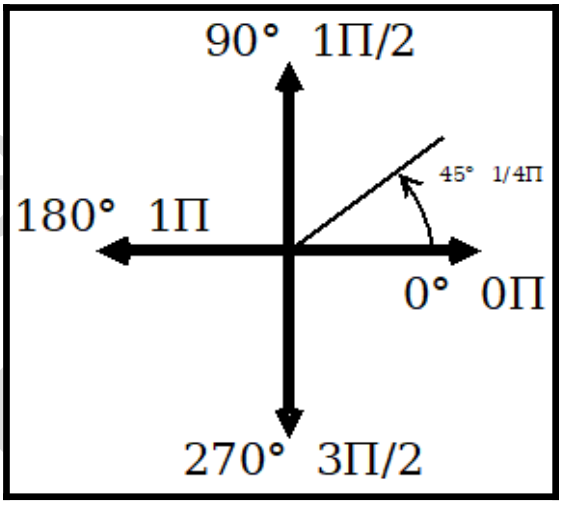

Illustration 39: Angles in Degrees and Radians

Because if this preference, there are two methods of math that will convert degrees to radians and radians to degrees.

```
1| import math
2|
3| a = 90
4| print(a,"degrees is",math.radians(a),"radians")
5|
6| b = math.pi
7| print(b,"radians is",math.degrees(b),"degrees")
```

```
90 degrees is 1.5707963267948966 radians
3.141592653589793 radians is 180.0 degrees
```

math.degrees(*angle_in_radians***)** **math.radians(***angle_in_degrees***)**	Methods of math
These methods convert an angle in degrees to radians and a degree in radians to degrees. The trigonometric methods in Python use radians, so these functions simplify use with degrees. https://docs.python.org/3/library/math.html#angular-conversion	

Basic Trigonometric Ratios

The trigonometric ratios of Sine, Cosine, and Tangent are helpful when we have the length of one side and a measured angle and want to know the length of the other sides. The math module includes methods called `math.sin()`, `math.cos()`, and `math.tan()` that implement these ratios. When calling these methods, remember to pass the angle's measurement in radians.

$$Sine\,\theta = \frac{Opposite}{Hypotenuse}$$

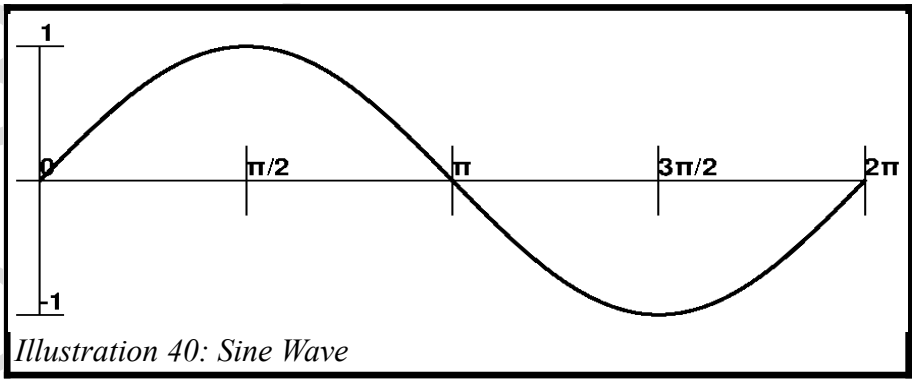

Illustration 40: Sine Wave

$$Cosine\,\theta = \frac{Adjacent}{Hypotenuse}$$

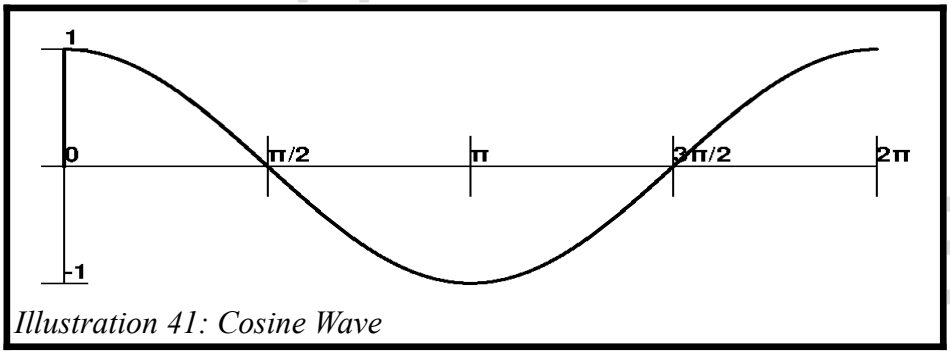

Illustration 41: Cosine Wave

$$Tangent\,\theta = \frac{Opposite}{Adjacent}$$

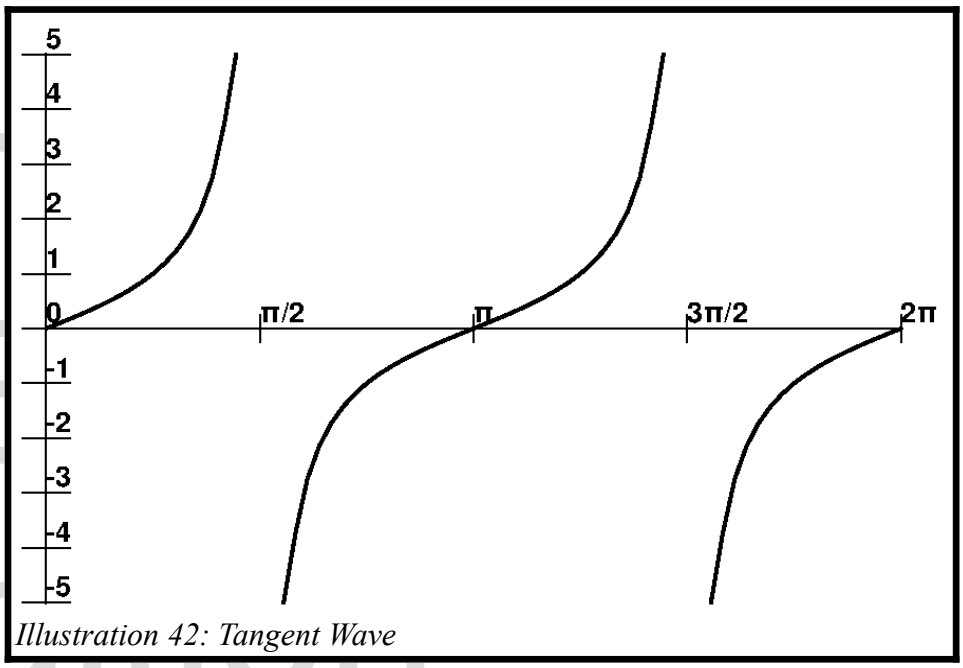

Illustration 42: Tangent Wave

```
 1| import math
 2|
 3| angle = float(input("Enter angle in degrees? "))
 4| hypotenuse = float(input("Enter length of hypotenuse? "))
 5|
 6| # sin = opp/hyp - solve for opposite
 7| opposite = math.sin(math.radians(angle)) * hypotenuse
 8|
 9| # cos = adj/hyp
10| adjacent = math.cos(math.radians(angle)) * hypotenuse
11|
12| print("the opposite side is", opposite, "long.")
13| print("the adjacent side is", adjacent, "long.")
```

```
Enter angle in degrees? 30
Enter length of hypotenuse? 10
the opposite side is 4.999999999999999 long.
the adjacent side is 8.660254037844387 long.
```

math.sin(*angle*) **math.cos(*angle*)** **math.tan(*angle*)**	Methods of math
Returns the ratio of the length of two sides of a right triangle, based on one of the	

angles. https://docs.python.org/3/library/math.html#trigonometric-function	

Inverse Trigonometric Ratios

The inverse trignometric methods ArcSine, ArcCosine, and ArcTangent are implemented with the `math.asin()`, `math.acos()`, and `math.atan()` methods. These allow us to find the angle's measurement in radians if we know it's sine, cosine, or tangent. They are very useful if we know the length of two sides.

$$\theta = ArcSine\left(\frac{Opposite}{Hypotenuse}\right)$$

$$\theta = ArcCosine\left(\frac{Adjacent}{Hypotenuse}\right)$$

$$\theta = ArcTangent\left(\frac{Opposite}{Adjacent}\right)$$

```
1| import math
2|
3| opposite = float(input("Enter length of opposite side? "))
4| adjacent = float(input("Enter length of adjacent side? "))
5|
6| a = math.atan(opposite/adjacent)
7|
8| print("the angle is", a, "radians.")
9| print("or", math.degrees(a), "degrees.")
```

```
Enter length of opposite side? 5
Enter length of adjacent side? 10
the angle is  0.4636476090008061 radians.
or 26.56505117707799 degrees.
```

`math.asin(`*`ratio`***`)`**
`math.acos(`*`ratio`***`)`**

`math.atan(ratio)`	Methods of math
Returns the angle (between 0 and pi) for the ratio. In some languages these functions are called the "inverse" functions because they do the opposite of sin, cos, and tan. https://docs.python.org/3/library/math.html#trigonometric-functions	

Summary

In this chapter we have seen the “math” module and some of its features. Math includes: mathematical constants like Pi and e; functions to font integers larger or smaller; logarithms and powers; and basic trigonometry.

Important Terms

- adjacent
- arccosine
- arcsine
- arctangent
- ceil
- common logarithm
- cosine
- degrees
- floor
- hypotenuse
- math
- natural logarithm
- opposite
- power
- radians
- sine
- tangent

Exercises

Here

Word Search

```
i i e l i a f h t c e i l t g o o
t m n t c · t s d p f l o o r s e
· c d d n o u a c n m n f p c i h
n d e g r e e s r a d i a n s n y
o p p o s i t e r t t a h r u e p
n a · d h e a a r c c o s i n e o
i r n w i e i e e n t s t g a a t
c o m m o n · l o g a r i t h m e
p o w e r u h t a n g e n t o m n
n o l s r m k a e c o s i n e a u
i o a r c t a n g e n t e t t s
f n e e t h t o a r c s i n e h e
h n p q s o · a o p e s k e o e p
e g e t t e r t · i n o o e u c t
m t a d j a c e n t g o v y f n d
n a t u r a l · l o g a r i t h m
i n t n n e h t d r s c e a m l n
```

adjacent, arccosine, arcsine, arctangent, ceil, common logarithm, cosine, degrees, floor, hypotenuse, math, natural logarithm, opposite, power, radians, sine, tangent

References

https://en.wikipedia.org/wiki/Pi
https://en.wikipedia.org/wiki/Logarithm
https://en.wikipedia.org/wiki/Trigonometry